RA 09

6.1 Introduction to Binary Relations

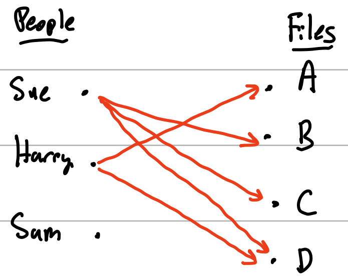
- A **binary relation** between two sets A & B is a subset R of $A \times B$.
    - For $a \in A$ and $b \in B$, $(a,b) \in R$ is denoted aRb.

- Arrow Diagram

  People = { Sue, Harry, Sam }

  Files = { A, B, C, D }

  Relation A : pAf is person p has access to file f



$A = \{(Sue,B), (Sue,C), (Sue,D), (Harry,A), (Harry,D)\}$

- Matrix Representation

  People = { Sue, Harry, Sam }

  Files = { A, B, C, D }

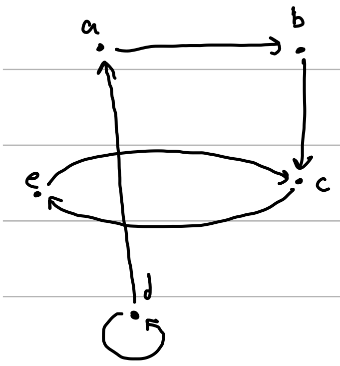
  Relation A : pAf is person p has access to file f

$$\begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} Sue \\ Harry \\ Sam \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$A = \{(Sue,B), (Sue,C), (Sue,D), (Harry,A), (Harry,D)\}$

- A <u>binary relation on a set</u> A is a subset of $A \times A$.
  - set A is called the <u>domain</u> of the binary relation.

Diagram Example



$A = \{a, b, c, d, e\}$

$R \subseteq A \times A$

$R = \{(a,b)\ (b,c)\ (e,c)\ (c,e)\ (d,a)\ (d,d)\}$

6.2 Properties of binary relations

- Suppose that R is a binary relation to set A. R is <u>reflexive</u> i.f.f for every $x \in A$, xRx.
  - "Every element must be related to itself"

- R is <u>anti-reflexive</u> i.f.f for every x in the domain of R, it is not true that xRx.

Symmetric binary relations

- Suppose that R is a relation of set A.
  - A relation is symmetric if for every pair of elements x and y in the domain, one of the following situations holds:
    - xRy and yRx are both true
    - Neither xRy nor yRx is true.

Anti-symmetric binary relations

- R is <u>anti-symmetric</u> if one of the following situations holds for every pair of distinct elements:
  - xRy, but yRx is not true
  - yRx, but xRy is not true.
  - Neither xRy nor yRx is true.

Transative Binary Relations

- R is <u>transative</u> i.f.f. for every three elements $x, y, z \in A$, if xRy and yRz, then xRz must also be true.

6.3 Directed graphs, paths, and cycles

- A <u>directed graph</u> consists of a pair (V, E).
  - V is the set of <u>vertices</u>, and E is a set of <u>directed edges</u> (also a subset of $V \times V$).
  - An individual element of V is a <u>vertex</u>.
    - pictured as a dot and is labeled
  - An edge $(u, v) \in E$ is pictured as an arrow: $u \longrightarrow v$
    - vertex u is the <u>tail</u> of the edge
    - vertex v is the <u>head</u>.
  - The <u>in-degree</u> of a vertex is the number of edges pointing into it.
  - The <u>out-degree</u> of a vertex is the number of edges pointing out of it.

- An open walk is a walk in which the first and last vertices are not the same.
- A closed walk is a walk is a walk in which the first and last vertices are the same.

- A trail is a walk in which no edge occurs more than once.
- A path is an open walk in which no vertex occurs more than once.
- A circuit is a closed trail.
- A cycle is a circuit of at least length 1 in which no vertex occurs more than once, except the first and last vertices which are the same.

6.4 Composition of relations

- The composition of relations R and S on set A is another relation on A, denoted $S \circ R$. The pair $(a,c) \in S \circ R$ i.f.f. there is a $b \in A$ such that $(a,b) \in R$ and $(b,c) \in S$.